**Mathematical Methods**

**In-class Investigation – Tables of Values**

**Time allowed: 40 minutes**

**Total marks: 40**

**Instructions: Answer all the questions in the spaces provided.**

**Scientific and Classpad calculators are allowed.**

**Show all working steps for part marks to be awarded.**

In this investigation, you learn to write the general formula from a table of values representing the following functions – linear, quadratic, reciprocal, exponential and cubic.

**Part A (6 marks)**

**1st differences constant:**

Consider the sequence 5, 11, 17, 23, … nth term is ***an + b***

n=1 n=2 n=3 n=4  n=1 n=2 n=3 n=4

5 11 17 23 a+b 2a+b 3a+b 4a+b

6 6 6 a a a













As there is a **constant difference** between the terms of this sequence, the terms represent a **linear** function, hence the formula is **y = ax + b**

Hence, for the table of values

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 |
| y | 5 | 11 | 17 | 23 |

Comparison of the numerical difference table with the algebraic difference table allows values to be assigned:

a = 6 and since a + b = 5, then b = -1

Hence the equation is y = 6x – 1

Using the method above, find the equation for each of the following tables of values:

1

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 |
| y | 10 | 13 | 16 | 19 |

2

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 |
| y | -7 | -11 | -15 | -19 |

3

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 |
| y | 1 | -4 | -9 | -14 |

**Part B (12 marks)**

**2nd differences constant:**

Consider the sequence 15, 35, 63, 99, …, nth term is ***an2 + bn + c***

As there are **two constant differences**, the terms represent a **quadratic** function, hence the formula is





















n=1 n=2 n=3 n=4  n=1 n=2 n=3 n=4

15 35 63 99 a+b+c 4a+2b+c 9a+3b+c 16a+4b+c

20 28 36 3a+b 5a+b 7a+b

8 8 2a 2a

Hence for the table of values

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 |
| y | 15 | 35 | 63 | 99 |

You can work up the first entries in the rows from the bottom row to the top row, equating and substituting to calculate the values of a, b and c. This gives a = 4, b = 8 and c = 3.

Hence the equation is 3

Using the method above, find the equation for each of the following tables of values:

1

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 |
| y | 2 | 9 | 22 | 41 |

2

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 |
| y | 2 | 4 | 4 | 2 |

3

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 |
| y | -8 | -9 | -6 | 1 |

**Part C (10 marks)**

**3rd differences constant:**

Consider the sequence 3, 4, 6, 12, 25, 48, …, nth term is ***an3 + bn2 + cn + d***

n=1 n=2 n=3 n=4 n=5 n=6 

3 4 6 12 25 48

1 2 6 13 23

1 4 7 10

3 3 3

























n=1 n=2 n=3 n=4

a+b+c+d 8a+4b+2c+d 27a+9b+3c+d 64a+16b+4c+d

7a+3b+c 19a+5b+c 37a+7b+c

12a+2b 18a+2b

6a













As there are **three constant differences**, the terms represent a **cubic** function, hence the formula is

Hence for the table of values

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 3 | 4 | 6 | 12 | 25 | 48 |

You can work up the first entries in the rows from the bottom row to the top row, equating and substituting to calculate the values of a, b, c and d. This gives a = , b = , c = 5 and d = 0.

Hence the equation is

Using the method above, find the equation for each of the following tables of values:

1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | -1 | 1 | 11 | 35 | 79 | 149 |

2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 5 | 5 | -1 | -19 | -55 | -115 |

**Part D (6 marks)**

In this **reciprocal** relationship, the product of coordinate pairs produces a constant result. Hence the formula is or where *k* is a constant.

Consider the table of values

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 2 | 4 | 8 | 16 |
| y | 8 | 4 | 2 | 1 |

Notice that the product of x and y is 16. Hence the equation for this set of values is or .

Find the equation for each of the tables of values:

1

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 |
| y | -12 | -6 | -4 | -3 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x |  |  | 1 | 2 |
| y | 9 |  | 3 | 1.5 |

2

3

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 2 | 5 | 10 |
| y | 2.5 |  |  |  |

**Part E ( 6 marks )**

An **Exponential** relationship produces a constant ***ratio*** pattern as opposed to a difference constant.

The general equation iswhere ***k*** and ***a*** are constants.

Consider the table of values

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| y |  | 1 | 2 | 4 | 8 | 16 | 32 |

**Ratio** 2 2 2 2 2

The ratio value is “a” and “k” is the y-value when x = 0. Thus and . Hence the equation for this set of values is

Find the equation for each of the table of values:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| y |  | 1 | 3 | 9 | 27 | 81 |

1

2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| y |  | 1 |  |  |  |  |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| y |  | 6 | 18 | 54 | 162 | 486 |

3